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Resonant features of planar Faraday metamaterial with high structural symmetry

Study of properties of a 4-fold array of planar chiral rosettes placed on a ferrite substrate

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Abstract. The transmission of electromagnetic wave through the planar chiral structure, loaded with the gyrotropic medium on the longitudinal magnetic field is studied. The frequency dependence of the metamaterial resonance and the angle of rotation of the polarization plane are obtained. We define both theoretically and experimentally the range of frequencies and magnetic fields, where the angle of polarization plane rotation for the metamaterial is essentially higher than for a single ferrite slab.

1 Introduction

Bulk chiral artificial structures [1], [2] (the classical object here is the spirally conducting cylinder), and planar chiral structures [3]-[6] (an array of metallic rosettes for instant) are quite interesting chiral metamaterials from both fundamental and application points of view. In particular, planar chiral metamaterials can be used successfully for design of magnetically controllable microwave devices [7], using the Faraday Effect, which reveals in the ferrite substrate.

Theoretical study of such ferromagnetic structures [7], as well as experimental [8] ones were carried out mainly in the centimeter wavelength range. However, in a more high-frequency band, which is quite important today from the application point of view, the experimental study of planar chiral ferromagnetic metamaterials with longitudinal magnetization was carried insufficiently.

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The purpose of this paper is to study the resonant properties of planar chiral gyrotropic metamaterial (an array of metallic rosettes placed on the ferrite substrate) depending on the value of static magnetic field (applied normally to the structure plane) and on the size of periodic cell of the planar chiral structure in the millimeter wavelength range. We consider metamaterial based on a 4-fold symmetry array consisted of planar chiral metallic elements. Planar chirality is introduced in the consideration with the object to complicate the shape of particles in the periodic cell of array. The using of complex shaped particles enables us to design resonant metamaterial which has small pitch in comparison with the wavelength.

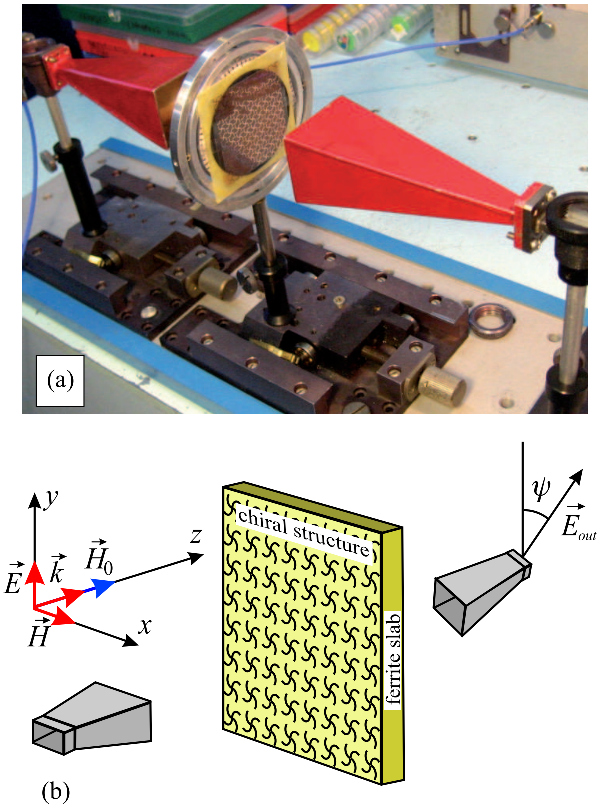

Fig. 1. (Color online) Experimental setup: (a) the overview; (b) the scheme of experiment.

2 The experimental technique and theoretical approach

The experimental setup [8] consists of the structure under study, which is placed between two matching rectangular horns (transmitting and receiving ones) fitted to the Vector Network Analyzer Agilent N5230A. Horns are situated on the axis passed normally to the plane of the structure (Fig. 1a). Using the Network Analyzer the S-parameters, namely S_{21} - the transmission coefficient for the structure in the frequency range 22-40 GHz can be detected and analyzed by the special computer program.

For measurements in a longitudinal static magnetic field the structure and horns are positioned between the poles of the electromagnet to provide the orientation of the components of AC field (E, H) and DC field (H_0) as it is shown in Fig. 1b. The electromagnetic parameters are controlled by computer. A more detailed technique of this fully automated experiment one can find in [8].

The metamaterial being under investigation is designed as a layered structure, which consists of ferrite plane-parallel slab with thickness of 0.5 mm (the brand of ferrite is L14H) and the planar chiral periodic structure. This structure is made of fiberglass ($\varepsilon' = 3.67, \tan\delta = 0.06$) with a thickness of 1.5 mm, one side of which is covered with copper foil. The foil side of this layered structure is patterned with a square unite cell periodic array consisted of planar chiral rosettes (see Fig. 2). The ferrite slab is leaned to this array of metallic elements. Thus, two

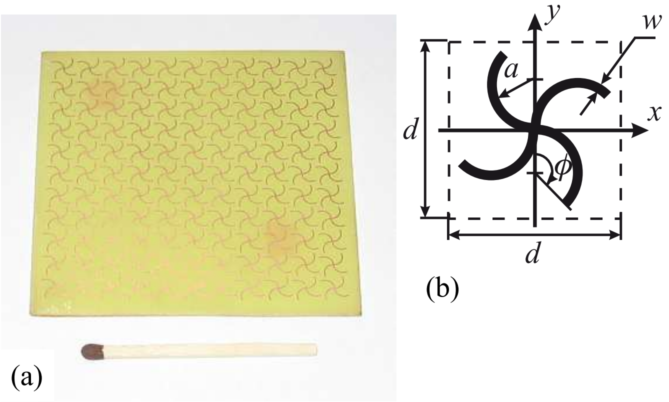

Fig. 2. (Color online) The periodic array of planar chiral elements placed on a dielectric substrate: (a) the photo of the planar chiral left-handed structure; (b) the square cell of the array (d is the period of the structure) with a metallic element shaped as the planar chiral right-handed rosette (a is the radius of leaf, ϕ is its angular size).

samples of gyrotropic planar metamaterial 60×60 mm^2 of either of the right-handed and left-handed kind, which differ by the period of the rosette array, have been produced. Sample 1 of both right-handed and left-handed kinds has the period $d = 5$ mm and the radius of arcs $a = 1.66$ mm, whereas Sample 2 has $d = 4$ mm and $a = 1.33$ mm. The angular size ϕ and the width w of copper strips that form the rosette, for both samples are identical being $\phi = 120$ deg and $w = 0.267$ mm, respectively.

We applied the 'resonant model' of 'saturated' ferrite [9], [10] to calculate the ferrite constitutive parameters for the static field more strong than the field of the saturation magnetization $4\pi M_S$, and the 'non-resonant model' of 'non-saturated' ferrite [11], [12] for the fields less than $4\pi M_S$.

For the field larger then $4\pi M_S$ we use common expressions for permittivity and permeability for z-axis biased ferrite [9], [10] assuming the ferrite material to be magnetically saturated and taking into account the dielectric and magnetic losses

$$\varepsilon_f = \varepsilon, \qquad \hat{\mu}_f = \begin{pmatrix} \mu & i\beta & 0 \\ -i\beta & \mu & 0 \\ 0 & 0 & \mu_z \end{pmatrix}, \tag{1}$$

where

$$\mu = 1 + 4\pi(\chi' - i\chi''), \quad \beta = 4\pi(K' - iK''), \quad \mu_z = 1, \tag{2}$$

$$\begin{gathered} \chi' = \omega_0\omega_m[\omega_0^2 - \omega^2(1-\alpha^2)]D^{-1}, \\ \chi'' = \omega\omega_m\alpha[\omega_0^2 + \omega^2(1+\alpha^2)]D^{-1}, \end{gathered} \tag{3}$$

$$\begin{gathered} K' = \omega\omega_m[\omega_0^2 - \omega^2(1+\alpha^2)]D^{-1}, \\ K'' = 2\omega^2\omega_0\omega_m\alpha D^{-1}, \end{gathered} \tag{4}$$

$$\begin{gathered} D = [\omega_0^2 - \omega^2(1+\alpha^2)]^2 + 4\omega_0^2\omega^2\alpha^2, \\ \omega_m = \gamma 4\pi M_S, \end{gathered} \tag{5}$$

ω_0 is the frequency of ferromagnetic resonance (FMR) and α is a dimensionless damping constant, γ is the gyromagnetic ratio, M_S is the saturation magnetization. We use the Gaussian system of units. Ferrite material of brand L14H is characterized by the following set of parameters: $\varepsilon = 13.2 - i0.0697$, $\alpha = 0.0285$, $\omega_m/2\pi = 14.2$ GHz. The value ω_m corresponds to the saturation magnetization field of $4\pi M_S = 4800$ Oe.

For fields smaller than $4\pi M_S$, the experiment is described well by the of non-resonant 'non-saturated' ferrite model [11], [12]. The elements of the tensor (1) of an un-

saturated ferrite represented by empirical expressions [12]:

$$\mu = \mu_{dem} + (1 - \mu_{dem})(M/M_S)^{3/2},$$
$$\mu_z = (\mu_{dem})^P, \quad P = (1 - M/M_S)^{5/2}, \tag{6}$$
$$\beta = -\gamma 4\pi M/\omega, \quad \mu'' = \mu_z'' = \beta'' = 0,$$

where M is the current value of the magnetization of the sample, μ_{dem} is the permeability of completely demagnetized ferrite, calculated on the basis of a two-domain model [11] for frequencies $\omega > \gamma(H_r + 4\pi M_S)$:

$$\mu_{dem} = \frac{1}{3} + \frac{2}{3}\sqrt{\frac{(\omega/\gamma)^2 - (H_r + 4\pi M_S)^2}{(\omega/\gamma)^2 - H_r^2}}. \tag{7}$$

Here, M_S is the saturation magnetization, H_r is the field matched to the remanent magnetization. For the used ferrite brand it is $H_r = 3500$ Oe. Components of the permeability tensor of ferrite versus static magnetic field strength are presented in Fig. 3 for the frequency $f = \omega/2\pi = 30$ GHz.

The frequency of FMR ω_0 for the ferrite slab is defined by the well-known formula for a thin slab magnetized normally to its plane [10]:

$$\omega_0 = \gamma|H_0 - 4\pi M_S|. \tag{8}$$

Dependence of FMR frequency versus static magnetic field strength is given in Fig. 4. Note that the formula (8) is a rigorous for the fields H_0 larger than $4\pi M_S$. For fields, which are less than $4\pi M_S$, the frequency of FMR may be somewhat lower due to the fact that the ferrite changes in the multidomain state and a violation of its magnetic order growth with decreasing of static field (see the dashed line in Fig. 4). On the same reason, the FMR linewidth should growth with the field decreasing.

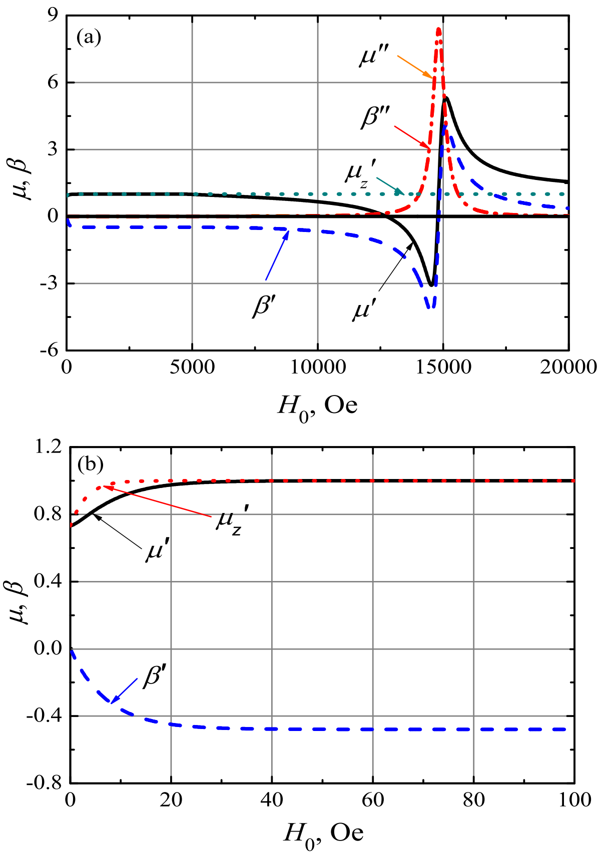

Fig. 3. (Color online) (a) Theoretical dependences of the components of permeability tensor for the thin ferrite slab versus the normally applied static magnetic field at $f = 30$ GHz; (b) the same dependences detailed for low strength of static field by non-resonant ferrite model.

As the field decreases below $4\pi M_S$ the domain structure appears in the ferrite and its magnetic state demonstrates a certain disorder. Note that in this case the values of the diagonal components of the $\hat{\mu}_f$, i.e. the value μ tends to permeability of completely demagnetized ferrite μ_{dem} (7). This value is not equal to zero (Fig. 3b). The latter is reasonable, because when domains disorder, then their contribution to the integral magnetization decrease. However the magnetization of each domain is a positive value, in spite of the external field is directed along the

domain magnetic moment or against it. Contributions to the diagonal components μ from all domains are added and it tends to some constant with field decreasing. A quite different behavior is observed for the off-diagonal component β. When field decreases, the domains, which magnetic moment is directed along the external field, and domains, which magnetic moment is directed opposite the field, give a different sign for the contribution to the β (the nonreciprocal Faraday effect). Thus, contributions of all domains to the off-diagonal components β are subtracted and β tends to zero with field decreasing. Note also that for the field less than $4\pi M_S$ the correct count of the magnetic disorder of domain structure in the ferrite should lead to the gradual change of the components μ and β.

The fields, intensities, and polarization characteristics of the electromagnetic waves diffracted by the array of rosette-shaped elements were calculated by the full wave method described in [7]. This approach is based on the method of moments for solution of vector integral equation for surface currents induced by the electromagnetic field on the array elements [13]. The equation was derived with boundary conditions that assume a zero value for the tangential component of the electric field on metal strips. In our calculations, we used the Fourier transformations of fields and surface current distributions.

3 Experiment and data analysis

First of all, let us mention that experimental study of transmission of normally incident wave through two kind of planar chiral arrays differed by sign of chirality was carried out in both cases of free standing arrays and arrays placed on ferrite substrate. We do not observe any difference in intensity of transmitted field and polarization transformations versus the sign of planar chirality of samples. Thus the experimental evidence of indistinguishability of these properties has been demonstrated between two enantiomorphous kinds of planar chiral samples consisted of right-handed and left-handed thin metallic rosettes in the case of normally incident wave. Before this property was argued theoretically in [6], [7].

Basing on the theoretical approach described above, the dependence of the metamaterial resonance peak frequency f_r for linearly y-polarized normally incident plane electromagnetic wave for the two values of the planar chiral structure period (d) on the DC magnetic field (Fig. 4) has been calculated. Besides that, the dependence of the FMR frequency on the DC magnetic field for the thin slab of ferrite used in experiments ($f_0(H_0) = \omega_0/2\pi$) is plotted in the same figure.

One can see that: (i) the variation of the resonant frequency of response peaks (df_r/dH_0) is as stronger as the frequency of the resonance peak is closer to the FMR frequency f_0. This fact is caused, obviously, that near the FMR the value of the real part of the diagonal component of the permeability μ considerably increases. In turn, μ is uniquely connected with the value of the resonant frequency related to array; (ii) in the range of magnetic fields from 12500 Oe up to 15000 Oe two resonant peaks i.e. two values of resonant frequency for the same value of the magnetic field are observed. Such scenario is caused

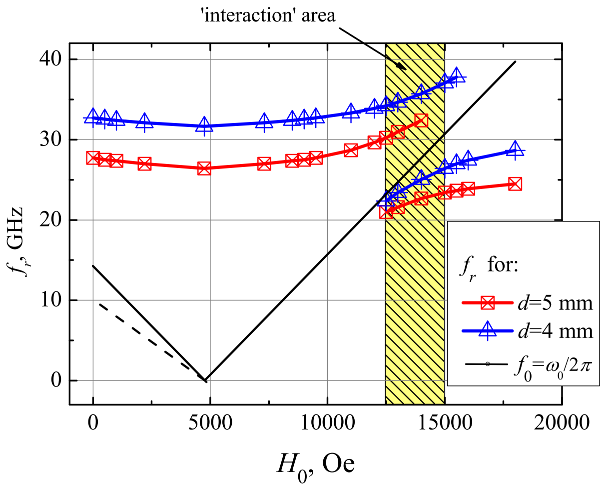

Fig. 4. (Color online) Theoretical dependence of the metamaterial resonance peak frequency on the magnetic field for two values of period of the planar chiral structure. The solid line denotes the calculated dependence of resonance frequency of the ferrite on the static field.

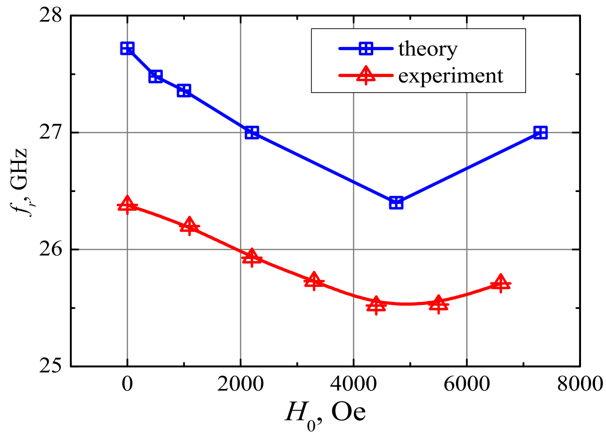

Fig. 5. (Color online) The dependence of the metamaterial resonance peak frequency on the static magnetic field for planar chiral structure $d = 5$ mm.

by the effect of resonance not only diagonal but also off-diagonal components of the permeability on the peak. In particular it is known [9], [10], that in the vicinity of FMR frequency the electromagnetic wave propagation constant for the longitudinal magnetization of the ferrite can acquire more than one value (in the given case, it is two). To be specific, let us call the area, where the resonant frequency of array and FMR frequency are close enough as 'interaction area'.

Comparison of experimental data and theoretical conclusions has been made in the field range 0-6500 Oe. In particular the qualitative agreement between experimental and calculated data for the dependence of metamaterial resonance peak frequency on the magnetic field (for the $d = 5$ mm) is occurred (Fig. 5). Note that the change of sign of df_r/dH_0 from negative to positive when the magnetic field exceeds the value corresponding to the saturation magnetization field ($4\pi M_S = 4800$ Oe), is related to the mentioned above effect, namely the presence of low-field mode (with $df_0/dH_0 < 0$) in the FMR spectrum [9] for fields less than $4\pi M_S$. However, as it was expected, the slope of the experimental frequency dependence of the metamaterial resonance peak on the magnetic field is a bit smaller than in the theory (due to the formation of magnetically disordered domain structure). The maximal value of frequency shift of metamaterial resonance peak of the magnetic field (triangle markers in Fig. 5) is about 900 MHz.

In order to verify the nonreciprocal properties of the metamaterials under study, the experimental analysis of electromagnetic wave transmission for the case where the angle between the plane of polarization of transmitting and receiving horn is $\psi = 45$ deg. It can be seen (Fig. 6) that both character and magnitude of the shift of metamaterial resonance peak frequency depend strongly on the

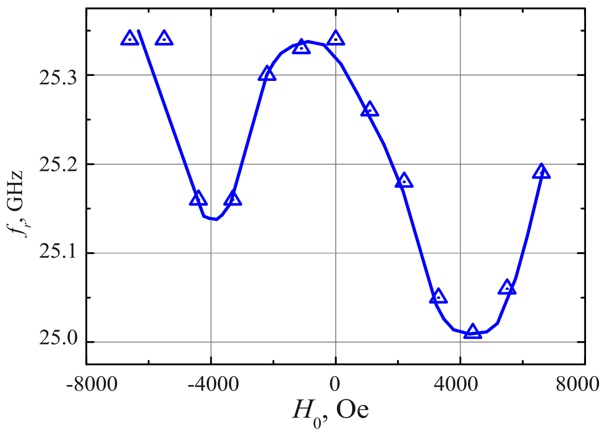

Fig. 6. (Color online) Experimental dependence of the metamaterial resonance peak versus the static magnetic field for the gyrotropic planar chiral structure $d = 5$ mm, for $\psi = 45$ deg.

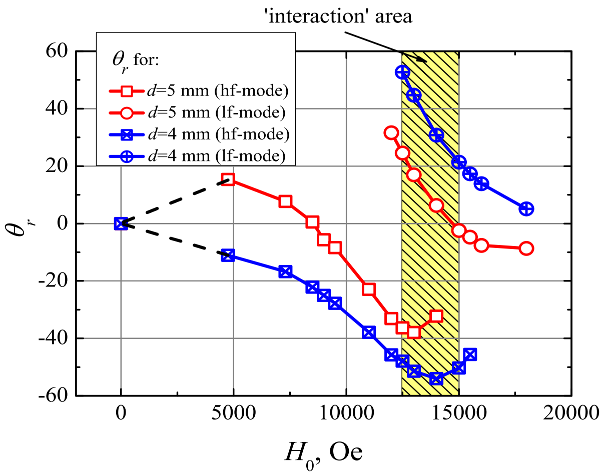

Fig. 7. (Color online) Theoretical dependences of the polarization rotation angle by the planar chiral structure loaded with ferrite slab versus the static magnetic field for two values of the structure period d.

static magnetic field direction. Let note, that for $\psi =$ 90 deg this dependence has the symmetric form as expected. Thus, the nonreciprocal properties of the investigated planar metamaterial are demonstrated.

For a more detailed study of the polarization properties of the metamaterial under study we have performed the experimental and theoretical analysis of the polarization plane rotation (more exactly, of the rotation of main axis of the polarization ellipse) of the wave transmitted through the structure with respect to the linear polarized wave incident on the structure. Theoretical dependences of the angle of polarization rotation for two metamaterial resonant modes on the magnetic field $\theta_r(H_0)$ for the planar chiral structure loaded with ferrite slab, and for two values of its period d are shown in Fig. 7.

The points marked by squares correspond to the high-frequency modes (hf-modes located to the left of dependency $f_0(H_0)$ in Fig. 4), and the points marked by circles correspond to a low-frequency modes (lf-modes located to the right of dependency $f_0(H_0)$).

Easy to see that for both modes while the field tends to zero, the rotation angle also decreases to zero. This fully coincides with used theoretical models of ferrite permeability (Fig. 3), where it was shown that the off-diagonal component β which is responsible for polarization rotation tends to zero with the field decreasing.

This occurs, as mentioned above, due to the compensation of the effect of multidirectional domains orientation on the rotation angle. However, let note, that in the 'interaction area' ($H_0 \approx 12500 - 15000$ Oe) polarization rotation angles increase drastically. It can be seen that for high-frequency modes (square markers) the maximum of θ_r reaches $\theta_r \approx -50$ deg. For low-frequency modes (circle markers), this dependence looks monotone (in the given fields), and reaches the maximum values $\theta_r \approx 50$ deg.

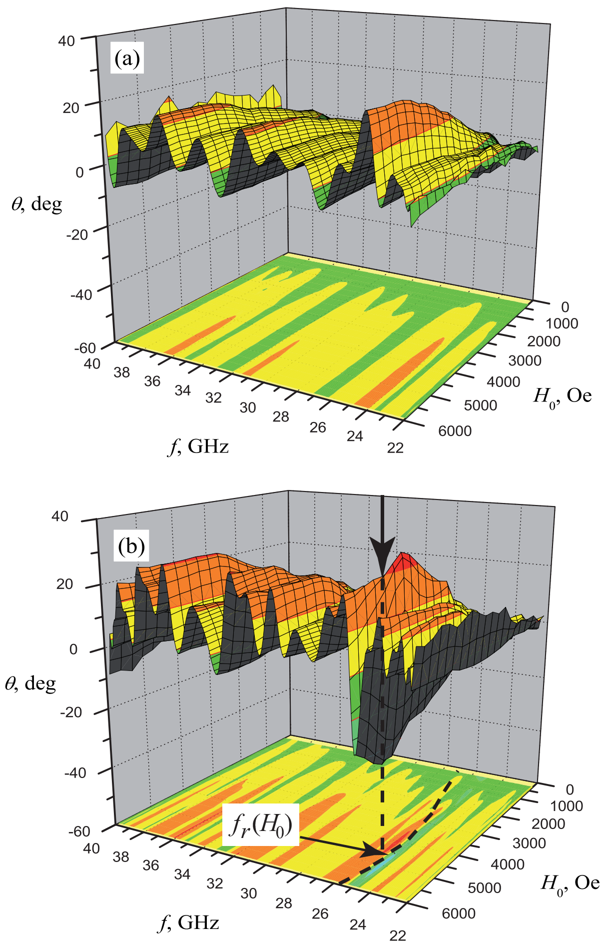

Fig. 8. (Color online) Experimental dependences of the polarization rotation angle θ as a function of frequency and static magnetic field for: (a) ferrite slab; (b) ferrite loaded planar chiral structure with period $d = 5$ mm.

Such resonant-like behavior of θ_r in the 'interaction' area is occurred obviously due to the large values of the off-diagonal components of the ferrite permeability (Fig. 3a) near the FMR.

The results of experimental verification $\theta_r(H_0)$ (Fig. 7) and $f_r(H_0)$ (Fig. 4) are summarized in Fig. 8. For the most clear demonstration of the effect of geometrical parameters on the metamaterial under study on the polarization processes experimental data are shown for: (i) the polarization rotation angle θ of linear polarized wave transmitting through a ferrite slab (Fig. 8a); (ii) the polarization rotation angle θ of linear polarized wave transmitting through planar chiral structure loaded with a ferrite slab in the case of period $d = 5$ mm (Fig. 8b).

One can see that the surface plotted for the ferrite slab (Fig. 8a) is much smoother than that one for the array structure loaded with ferrite slab (Fig. 8b). The monotonic growth of θ from 0 deg to 15 deg with increasing field from 0 Oe to 6500 Oe for all frequencies is occurred for the ferrite slab. A presence of moderate peaks is caused by the impossibility to provide the perfect matching of elements of the experimental setup. Also, for the planar chiral array loaded with ferrite slab, a monotonic growth of θ on the field takes place. However, near the frequency of the array resonance ($f_r = 25.5 - 26.5$ GHz (Fig. 5)), this dependence acquires a pronounced resonant character, and for $\theta \to \theta_r$ achieves significantly higher values than for the ferrite slab (up to $\theta \geq 45$ deg).

It can be seen that the value θ_r (Fig. 8b) also depends on the magnetic field, and the maximum of θ_r is observed at $H_0 \approx 4800$ Oe (i.e. in the transition area from saturated ferrite model to unsaturated one). In this region the real part of permeability has extreme (Fig. 3a), which explains the extreme in dependency $\theta_r(H_0)$.

Theoretical and experimental curves for the chiral structure loaded with the ferrite slab are similar in shape and exhibit a character extreme in the vicinity of the fields close to the saturation magnetization, as it is expected from the general representations.

The important feature is that the planar chiral Faraday metamaterial i.e. the resonant planar array loaded with ferrite slab manifests much large sensitivity of the polarization properties to the static magnetic field than the same ordinary slab of ferrite. This phenomenon can be explained by the fact that the resonant character of the magnetic permeability component of ferrite (or their strong frequency dispersion) is applied on the resonant character of oscillations in the planar chiral structure (strong frequency dispersion of the effective material parameters of the chiral structure), that takes place in the 'interaction area'. Note that a similar situation, known as the amplification of the Faraday effect have been found by the authors in the millimeter wave range before, but in more simple resonator structures (the open resonator [14], the photonic crystal [8]). However, in the case considered here, we are dealing with the structure being planar resonant metamaterial that promises the similar effect but with using very thin structure. The need resonant properties of thin metamaterial slab are imparted complex shaped metallic rosettes. The complex shape of array particles enables us to achieve resonant response of the structure in the wavelength less than pitch of the array. The 4-fold symmetry planar chiral rosettes are chosen to clear the way for design polarization insensitive array structure at least at normal incidence. Thus we can produce sub-wavelength resonant structures suitable for promising applications as planar metamaterial which is controllable by static magnetic field.

4 Conclusion

The transmission of electromagnetic waves of millimeter range through the layered metamaterial formed by the resonant planar chiral structure loaded with the gyrotropic medium has been studied both experimentally and theoretically. Namely: (i) the dependence of frequency of the metamaterial resonant peak response and the angle of polarization rotation on the longitudinal static magnetic field are detected, and a satisfactory agreement between the theory and experiment is demonstrated; (ii) the range of frequencies and magnetic fields is defined where the angle of polarization rotation by the metamaterial appears essentially higher than related to a single ferrite slab; (iii) the usage of arrays with high structural symmetry based on planar chiral particles enables additional means to produce sub-wavelength resonant metamaterials, which have small size of the periodic cell and controllable properties by static magnetic field.

References

1. B. Z. Katsenelenbaum, E. N. Korshunova, A. N. Sivov, and A. D. Shatrov, Physics-Uspekhi. **40,** 1149 (1997).
2. A. Serdyukov, I. Semchenko, S. Tretyakov, and A. Sihvola, *Electromagnetics of Bi-anisotropic Materials: Theory and Applications*, (Gordon and Breach Science Publishers, Amsterdam, 2001).
3. L. R. Arnaut, J. Electromagn. Waves Appl. **11,** 1459 (1997).
4. S. L. Prosvirnin, *in Proc. 7-th Intern. Conf. Complex Media, Bianisotropics-98, Braunschweig, Germany*, 1998, pp. 185–188.

5. S. Zouhdi, G. E. Couenon, and A. Fourrier-Lamer, IEEE Trans. Antennas Propagat. **47,** 1061 (1999).

6. S. L. Prosvirnin and N. I. Zheludev, J. Opt. A: Pure Appl. Opt., **11,** 074002 (2009).

7. S. L. Prosvirnin and V. A. Dmitriev, Eur. Phys. J. Appl. Phys., **49,** 33005 (2010).

8. A. A. Girich, S. Y. Polevoy, S. I. Tarapov, A. M. Merzlikin, A. B. Granovsky, and D. P. Belozorov, Solid State Phenomena, **190,** 365 (2012).

9. E. Collin, *Foundation for Microwave Engineering*, (Wiley-Interscience, New York, 1992).

10. A. G. Gurevich, *Ferrites at Microwave Frequencies*, (Heywood, London, 1963).

11. E. Schlomann, J. Appl. Phys. **41,** 204 (1970).

12. J. Green and F. Sandy, IEEE Microwave Theory Tech. **22,** 641 (1974).

13. S. L. Prosvirnin, J. Commun. Techn. Electron. **44,** 635 (1999).

14. S. I. Tarapov, Yu. P. Machekhin, and A. S. Zamkovoy, *Magnetic Resonance for Optoelectronic Materials Investigating*. (Collegium, Kharkov, 2008).